MID TERM REPETITION

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INF2310 - Digital Image Processing

Department of Informatics
The Faculty of Mathematics and Natural Sciences
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TODAY'S LECTURE

- Optical systems
  - Rayleigh criterion
  - Object-image relation

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- Neighbourhood operations and filters
  - Convolution
  - Low-pass filtering
  - High-pass filtering
  - Edge-detection filtering

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  - Edge-detection filtering
- Color imaging
  - Color models and spaces
  - Color images

OPTICAL SYSTEMS

POINT SOURCE

- We will study the light ray trajectory from a point source.
- A point will be imaged as a ”blurred” version of itself.
- The reason for this is two unrelated phenomena: *abberation* and *diffraction*.

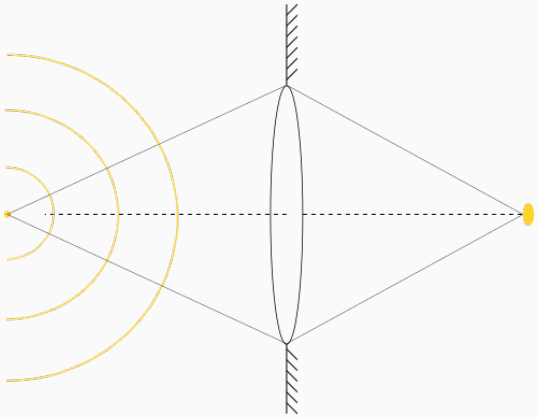
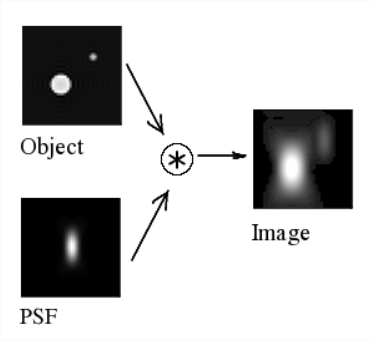


Figure 1: Light wave propagation from point source through circular aperture.

POINT SPREAD FUNCTION

- The point spread function (psf) describes the response from of an imaging system on a point source.
- The resulting image is obtained by convoluting the source with the point spread function.
- Can be used to measure the blurring of a point object, and is a measure of the quality of the imaging system.



OPTICAL ABBERATION

- Distortion in the image formed by an optical system.
- Arise because of limitations in optical components such as lenses and mirrors.
- There are many types of optical abberations, and they can to a large extent be repaired.

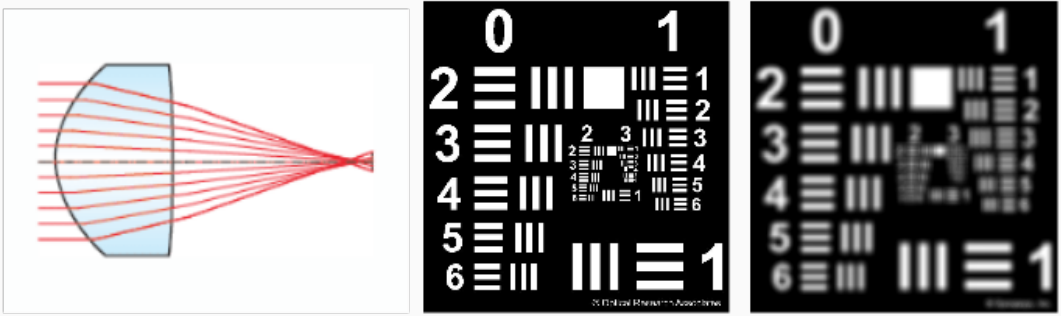


Figure 3: Spherical abberation. Rays from the edge of the lens focus points along a line in stead of at a single point. (Source: `http://www.edmundoptics.com`)

DIFFRACTION

- Caused by wavefronts of propagating waves bending in the neighborhood of obstacles.
- A fundamental limit in imaging.
- Possible to ”avoid” this limit with super-resolution imaging.

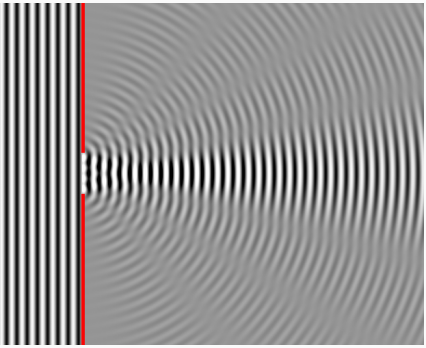


Figure 4: Illustration of diffraction pattern from a slit of width four wavelengths with an incident plane wave. (Source: By Dicklyon at English Wikipedia - Transferred from en.wikipedia to Commons by Shizhao using CommonsHelper., Public Domain, https://commons.wikimedia.org/w/index.php?curid=5699291)

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- A system is said to be *diffraction limited* if diffraction is the dominating the point spread function. That is, if the spatial resolution is as good as the instrument's theoretical limit.
- The resulting image from a point source imaged from a diffraction limited optical instrument is termed a *Airy disk*.

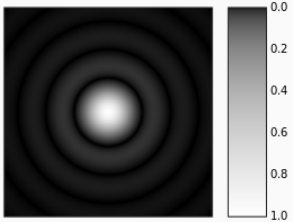

Figure 5: Airy disk

SPATIAL RESOLUTION

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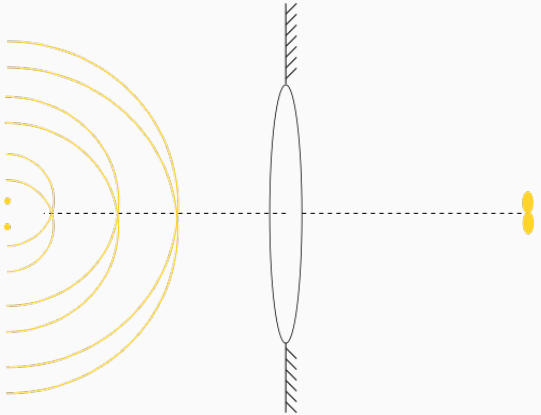
- Because of the blurring, described by the point spread function, there is a limit to the resolving ability of an image.



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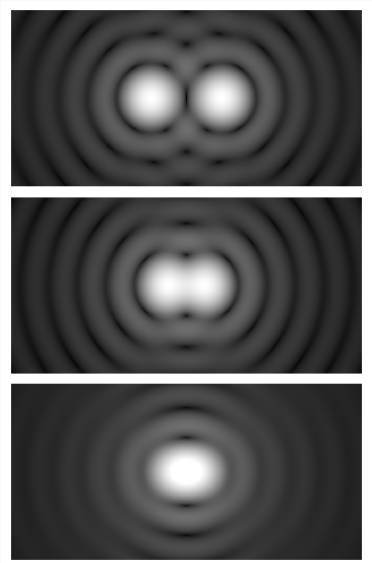
- The Rayleigh criterion: Two objects are just dissolved when the mode of the intensity function from one of the objects overlaps with the first zero of the second object's intensity function.



FRAUENHOFER DIFFRACTION PATTERN FOR A CIRCULAR APERTURE

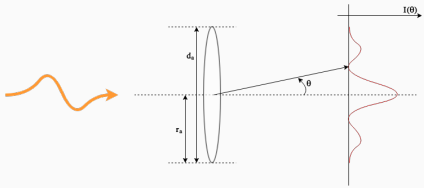


Figure 8: Diffraction pattern from circular aperture.

$$I(\theta) \propto \left[\frac{2J_1(kr_a \sin\theta)}{kr_a \sin\theta}\right]^2 \tag{1}$$

- J_1: A *first order Bessel function of the first kind*
- $k = \frac{2\pi}{\lambda}$: Wave number of the propagating light
- λ: Light wave length.
- r_a: Aperture radius.
- θ Angle to the first zero.

RAYLEIGH CRITERION EQUATION

- The intensity function in eq. (1) is zero when the Bessel function is zero.

[1] Bessel functions are solutions to a particular set of differential equations, and the mathematics is somewhat complicated. But feel free to stydy them on your own if you want to understand how the roots are found.

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- The intensity function in eq. (1) is zero when the Bessel function is zero.
- The first zero of a Bessel function[1] $J_1(x)$ occurs at $x \approx 3.8317$,

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- Thus

$$kr_a \sin\theta \approx 3.8317$$

or

$$\sin\theta \approx 1.22\frac{\lambda}{d_a} \tag{2}$$

which is famously known as the *Rayleigh criterion*.

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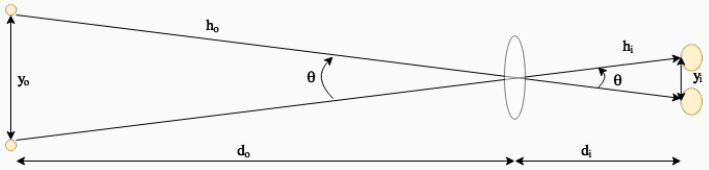


Figure 9: Schematic figure of distances.

For very small θ, $\sin\theta \approx \theta$. We can then approximate eq. (2) to

$$\theta \approx 1.22\frac{\lambda}{d_a}$$

and furthermore, $h_o \approx d_o$, yielding

$$y_o \approx 1.22\frac{\lambda}{d_a}d_o$$

This is the smallest distance we are able to dissolve in an image.

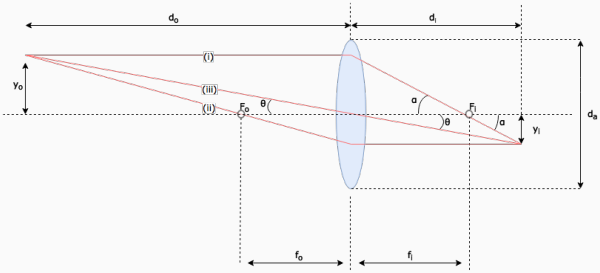
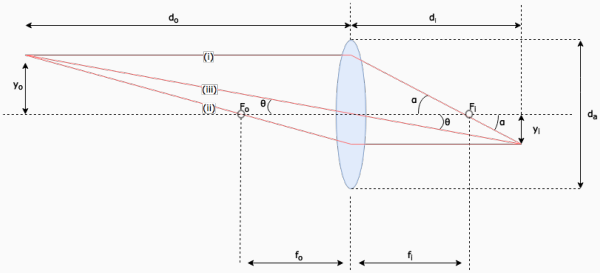


Figure 10: Idealised imaging system

(i) An incident ray which is parallel to the optic axis is refracted through the imaging focal point F_i.



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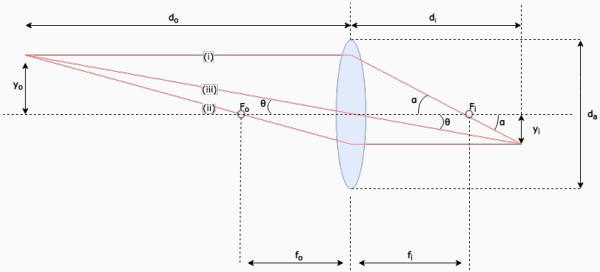
(i) An incident ray which is parallel to the optic axis is refracted through the imaging focal point F_i.

(ii) An incident ray which passes through the object focal point F_o, is refracted parallel to the optic axis.

PARAXIAL RAY APPROXIMATION



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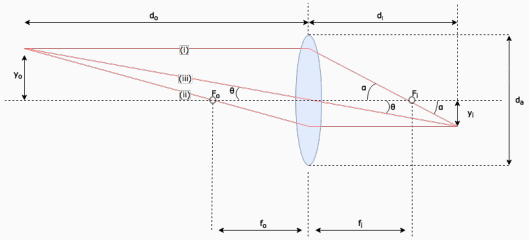
(i) An incident ray which is parallel to the optic axis is refracted through the imaging focal point F_i.

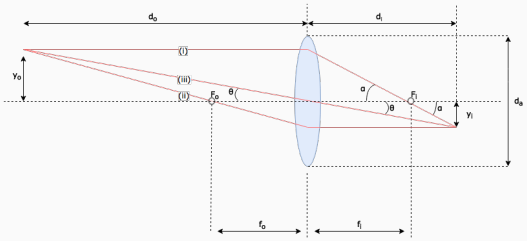
(ii) An incident ray which passes through the object focal point F_o, is refracted parallel to the optic axis.

(iii) An incident ray which passes through the optic center is refracted with the same refraction angle as incident angle (will not change direction).



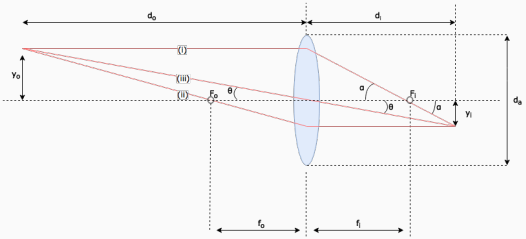
The rays following the respective patterns are illustrated in fig. 10. From (iii), we see that the angle θ is the same, and therefore

$$\frac{y_o}{d_o} = \frac{y_i}{d_i}. \tag{3}$$



Likewise, from the two equal angles α on ray (i), we get that

$$\frac{y_o}{f_i} = \frac{y_i}{d_i - f_i}. \tag{4}$$

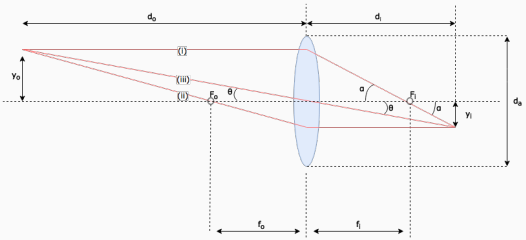


Rearranging eq. (3) yields

$$\frac{y_o}{y_i} = \frac{d_o}{d_i}$$

which, when combined with the relation in eq. (4) yelds

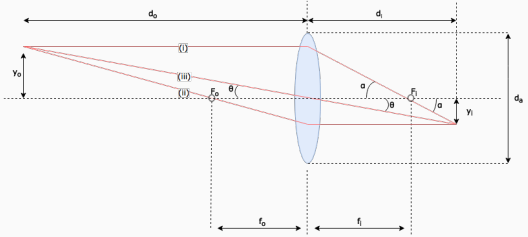
$$\frac{d_o}{d_i} = \frac{f_i}{d_i - f_i}. \tag{5}$$



Equation (5) can be rearranged to the familiar form

$$\frac{1}{f_i} = \frac{1}{d_i} + \frac{1}{d_o}. \tag{6}$$

Equation (6) is sometimes referred to as the *thin lens equation*.



Using eq. (3), we get that

$$\frac{1}{f_i} = \frac{1}{d_o}\left(\frac{y_o}{y_i} + 1\right),$$

which we can rearrange to

$$y_i = \frac{y_o f_i}{d_o - f_i}$$

This gives us (for our small, idealised imaging system) the relation between the actual size in the object plane y_o and the observed size in the imaging plane y_i.

SPATIAL FILTERING

In image analysis[1], convolution is a binary operation, taking an image f and a filter (also an image) w, and producing an image g. We use an asterisk $*$ to denote this operation

$$f * w = g.$$

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For an element $[x, y]$, the operation is defined as

$$g[x, y] = (f * w)[x, y] \coloneqq \sum_{s=-S}^{S} \sum_{t=-T}^{T} f[x - s, y - t] w[s, t].$$

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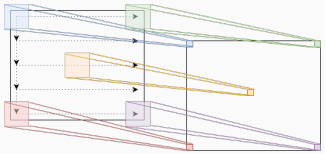

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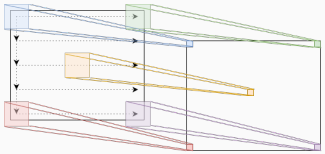

Also, check out this nice interactive visualization:
`http://setosa.io/ev/image-kernels/`

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Full: We get an output response at every point of overlap between the image and the filter.

Same: The origin of the filter is always inside the image, and we *pad* the image in order to preserve the image size in the result.

Valid: We only record a response as long as there is full overlap between the image and the *whole* filter.

FULL MODE — EXAMPLE

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \overset{\text{full}}{*} \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.3 & 0.6 & 0.9 & 0.7 & 0.4 \\ 0.6 & 1.4 & 2.4 & 3. & 2.2 & 1.2 \\ 1.5 & 3.3 & 5.4 & 6.3 & 4.5 & 2.4 \\ 2.7 & 5.7 & 9. & 9.9 & 6.9 & 3.6 \\ 2.2 & 4.6 & 7.2 & 7.8 & 5.4 & 2.8 \\ 1.3 & 2.7 & 4.2 & 4.5 & 3.1 & 1.6 \end{bmatrix}$$

VALID MODE — EXAMPLE

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \underset{*}{\overset{\text{valid}}{}} \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} = \begin{bmatrix} 5.4 & 6.3 \\ 9. & 9.9 \end{bmatrix}$$

Can also pad with other constant values than 0.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \xrightarrow{\text{zero}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 0 \\ 0 & 5 & 6 & 7 & 8 & 0 \\ 0 & 9 & 10 & 11 & 12 & 0 \\ 0 & 13 & 14 & 15 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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- An example is a 5×5 mean filter

$$\frac{1}{5}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} * \frac{1}{5}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

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$$f * w = f * (w_V * w_H) = (f * w_V) * w_H$$

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- As can be seen, we can now do two convolutions with a 1D filter, in stead of one convolution with a 2D filter, that is

$$g = f * w, \quad \text{2D}$$

vs.

$$h = f * w_V \quad \text{1D}$$

$$g = h * w_H \quad \text{1D}$$

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So the speed up should be linear in L.

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Feauture detection filters
: Used for detection of features in an image. Features such as edges, corners and texture.

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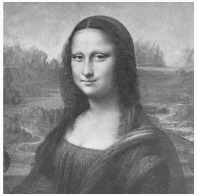
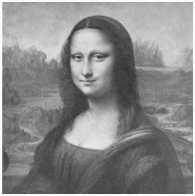
  - Large filter: Loss of details, more blurring.
  - Small filter: Preservation of details, less blurring.

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad \frac{1}{49}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

MEAN VALUE FILTER — EXAMPLE



(a) Original



(b) 3×3

(c) 9×9

(d) 25×25

Figure 11: Gray level Mona Lisa image filtered with a mean value filter of different sizes.

- For integer valus x, y, let

$$w[x, y] = A \exp\left\{-\frac{x^2 + y^2}{2\sigma^2}\right\}$$

where A is usually such that $\sum_x \sum_y w[x, y] = 1$.

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- This is a *non-uniform low-pass filter*.
- The parameter σ is the standard deviation and controls the amount of smoothing.
  - Small σ: Less smoothing
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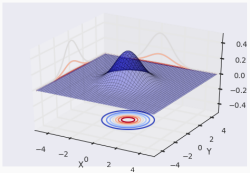
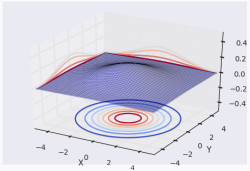
- A Gaussian filter smooths less than a uniform filter of the same size.



(a) $\sigma = 1$



(b) $\sigma = 2$

Figure 12: Continuous bivariate Gaussian with different σ. Implementation can be found at `https://ojskrede.github.io/inf2310/lectures/week_06/`

MEDIAN FILTERING

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HIGH-PASS FILTERING — OVERVIEW

- Filters that let through high frequencies, and damps or block low frequencies.

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- Noise is amplified.

- The convolution mask we have investigated can be rewritten as "high-pass = k * (original - low-pass)", where $k = 9$

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} = 9 \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right)$$

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- With this, our image improvement strategy can be viewed as:
  1. Low pass filtering
  2. Subtract the low-pass filtered image from the original
  3. Add k times the difference from point 2. to the original image.
- The described method is called *highboost-filtering*.

UNSHARP MASKING AND HIGHBOOST-FILTERING

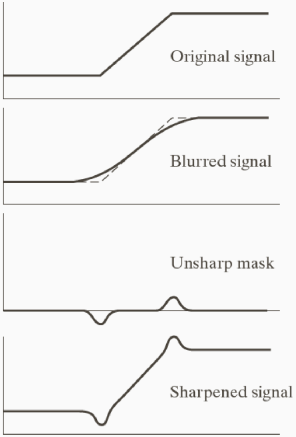


Figure 13: Schematic illustration of image sharpening on a 1D ramp.

UNSHARP MASKING AND HIGHBOOST-FILTERING

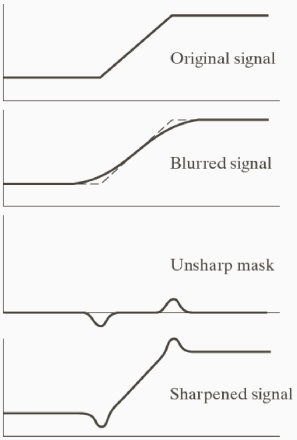
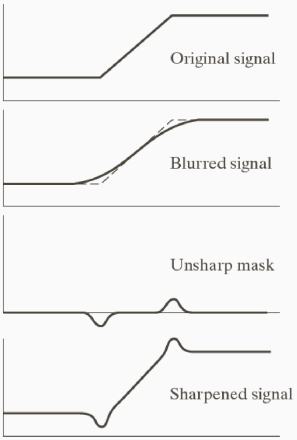


Figure 13: Schematic illustration of image sharpening on a 1D ramp.

- Given an image.
- Blur the image with a low-pass filter.

UNSHARP MASKING AND HIGHBOOST-FILTERING



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- Given an image.
- Blur the image with a low-pass filter.
- Compute the difference $original - blurred$

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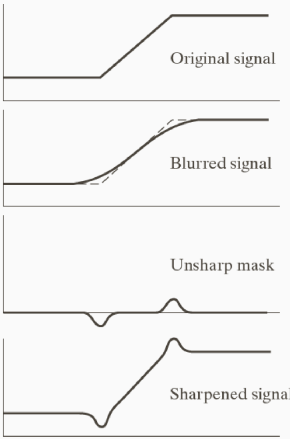

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- $sharpened = original + k(original - blurred)$

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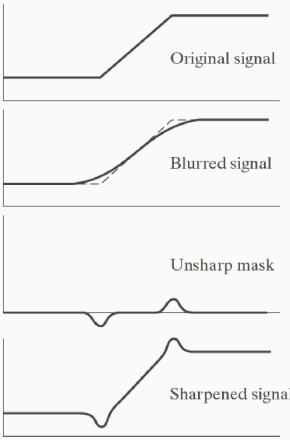

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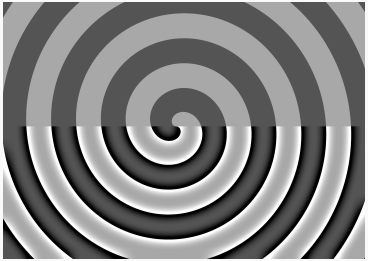
- Given an image.
- Blur the image with a low-pass filter.
- Compute the difference $original - blurred$
- $sharpened = original + k(original - blurred)$
- k is a positive constant.
  - $k = 1$: Unsharp masking
  - $k > 1$: Highboost-filtering

EXAMPLES



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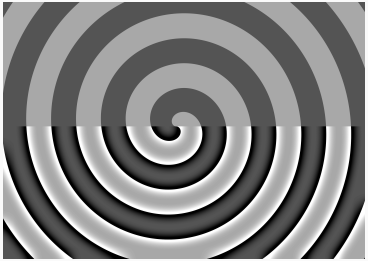

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- Biological visual systems are based on edge-detection.
- Such systems often works in parallel and/or sequential:
  - Local areas is treated independently
  - Local results can be dependent on earlier results

- The gradient points in the direction where the function ascends the most. We call this the *gradient angle*

$$\theta_g = \arctan\left(\frac{g_y}{g_x}\right)$$

SUMMARY — GRADIENT ANGLE AND GRADIENT MAGNITUDE

- The gradient points in the direction where the function ascends the most. We call this the *gradient angle*

$$\theta_g = \arctan\left(\frac{g_y}{g_x}\right)$$

- The ascent magnitude at this point is called the *gradient magnitude* and is

$$\begin{aligned} |\nabla f| &= \frac{\partial f}{\partial r}(\theta_g) \\ &= \sqrt{g_x^2 + g_y^2}. \end{aligned}$$

- Where

$$g_x = \frac{\partial f}{\partial x}, \text{ and } g_y = \frac{\partial f}{\partial y}$$

are the components of the gradient of f, ∇f.

DIGITAL GRADIENT APPROXIMATIONS

- We will approximate the gradient at some pixel (i, j) by the difference between pixels in the neighbourhood.

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- The convolution result $g_x[i, j]$ and $g_y[i, j]$ will then approximate the gradient at (i, j) in the vertical and horizontal direction, respectively.
- In the next slides, we will introduce several different gradient operators. Note that we write convolution filters, while G&W use correlation filters, and they are therefore rotated 180 degrees compared to ours.

Increasing noise robustness: 2D gradient operators

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- Then apply the desired gradient filter.
- We can utilize the separability of filters, and create new more noise-robust filters.

$$h_x = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix},$$

PREWITT OPERATOR

$$h_x = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix},$$

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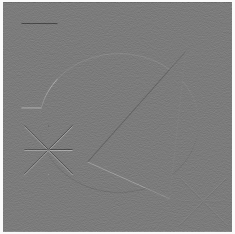
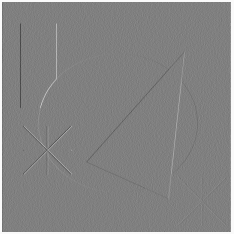
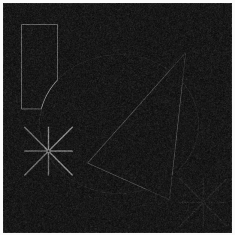
3x3 Sobel gradient operators



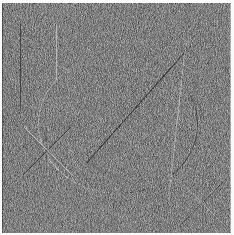
(a) g_x



(b) g_y

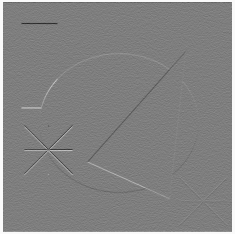


(c) $|\nabla f|$

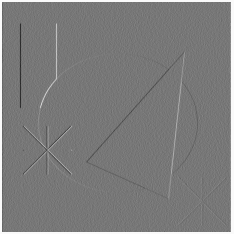


(d) θ

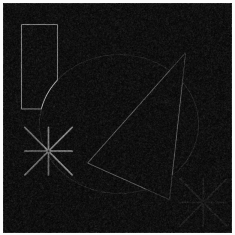
5x5 Sobel gradient operators



(a) g_x



(b) g_y



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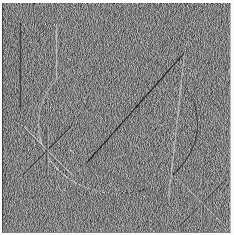

(d) θ

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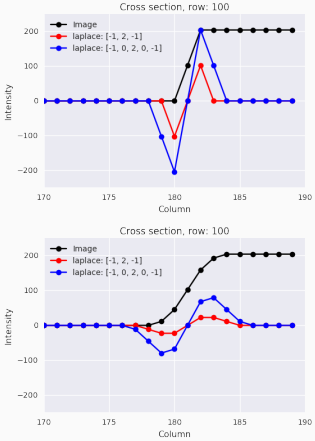
- Has two extrema for each edge, which is why we used it for image improvement earlier.
- If we use the location of the zero crossing, we can get a thin, well defined location of the edge.



2D LAPLACE OPERATOR

- Use the 1D Laplace approximation in both directions

$$
\begin{aligned}
-\nabla^2 f &= -\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2} \\
&\approx -(f[x+1,y] - 2f[x,y] + f[x-1,y]) - (f[x,y+1] - 2f[x,y] + f[x,y-1]) \\
&= -f[x+1,y] - f[x,y+1] + 4f[x,y] + f[x-1,y] + f[x,y+1]
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\end{aligned}
$$

- This can be computed by convolving the image with the following filter

$$
\begin{bmatrix}
0 & -1 & 0 \\
-1 & 4 & -1 \\
0 & -1 & 0
\end{bmatrix}
$$

FULL 3X3 LAPLACE OPERATOR

- If we, in addition, use the 1D Laplace approximation on the diagonal directions, we get the following filter

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

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- This convolution filter is the same as we used earlier in this slides for point detection and image sharpness enhancement.

COLOR IMAGING

THE COLOR OF AN OBJECT

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  - The spectral distribution of the reflected light.
- The reflection properties are determined by
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THE COLOR OF AN OBJECT

- The color we percieve in an object is determined by the nature of the light reflected by the object.
- Therefore, the color of an object is determined by
  - The light hitting the object.
  - How much light is reflected and absorbed.
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- Together, this determines what wavelengths are reflected, absorbed or transmitted.
- How we actually percieve color is complicated. Two complementary theories: *Trichromacy* and *Color opponency*.

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- The chromaticity determines both the dominating wavelength and the saturation of the color.
- Chromaticity and brightness together fully describes a color.
- For instance, different graylevels have the same chromaticity, but different intensity.

Figure 17: Chromaticity modeled as a unit polar coordinate chart (ρ, θ) where fully saturated colors are at the boundary $(\rho = 1)$, and the angle θ represent the wavelength, and thereby the hue.

ADDITIVE COLOR MIXING

- Mixing light of two or more colors

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- Utilized in TV and Computer monitors
- Difference between a mix of red and green (yellow) and yellow light (wavelength at about 580 nm), but we detect no difference.

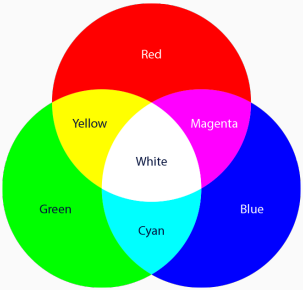

Figure 18: Additive mixing

SUBTRACTIVE COLOR MIXING

- Mixing filters or pigments that absorbs and reflect different colors.

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- Subtractive primaries: Yellow, Cyan, Magenta
- Subtractive secondaries: Red, Green, Blue
- Mixing of paint is subtractive (although, you probably used a RYB color model in kindergarden).

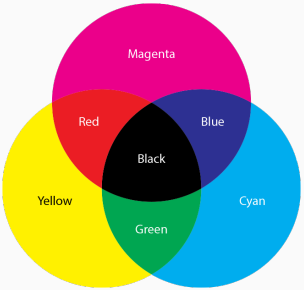

Figure 19: Subtractive mixing

RGB COLOR MODEL

- An additive color model where the primaries red, green and blue, are added together to form a wide variety of colors.

[1]By Gona.eu - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=6223756
[2]By SharkD - Own work, GFDL, https://commons.wikimedia.org/w/index.php?curid=3375025

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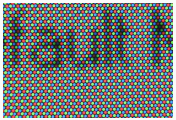
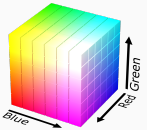
- Typical input devices are video cameras, digital cameras and image scanners.
- Typical output devices are TV, computer and mobile displays.

- Organized in a unit cube with coordinates (r, g, b), where $(0, 0, 0)$ is black and $(1, 1, 1)$ is white.
- A typical 24 bit display has 256 values for each channel.



(a) CRT monitor [1]



(b) RGB color cube [2]

[1] By Gona.eu - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=6223756
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CMYK COLOR MODEL

- **C**yan, **M**agenta, **Y**ellow, **K**ey (black)

[1] By BenRG and cmglee - http://commons.wikimedia.org/wiki/File:CIE1931xy_blank.svg, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=32158329

CMYK COLOR MODEL

- **C**yan, **M**agenta, **Y**ellow, **K**ey (black)
- CMYK (in $[0,1]$) to RGB (in $[0,1]$):

$$
\begin{aligned}
R &= (1-C)(1-K) \\
G &= (1-M)(1-K) \\
B &= (1-Y)(1-K)
\end{aligned}
$$

- RGB (in $[0,1]$) to CMYK (in $[0,1]$):

$$
\begin{aligned}
K &= 1 - \max\{R,G,B\} \\
C &= (1-R-K)/(1-K) \\
M &= (1-G-K)(1-K) \\
Y &= (1-B-K)(1-K)
\end{aligned}
$$

[1] By BenRG and cmglee - http://commons.wikimedia.org/wiki/File:CIE1931xy_blank.svg, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=32158329

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- The CMYK-model is subtractive

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$$K = 1 - \max\{R, G, B\}$$
$$C = (1 - R - K)/(1 - K)$$
$$M = (1 - G - K)(1 - K)$$
$$Y = (1 - B - K)(1 - K)$$

- The CMYK-model is subtractive
- RGB is common in display monitors, CMYK is common in printers.

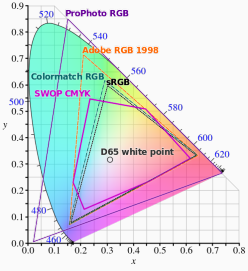

Figure 21: [1] Collection of gamuts

[1] By BenRG and cmglee - http://commons.wikimedia.org/wiki/File:CIE1931xy_blank.svg, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=32158329

HUE, SATURATION, INTENSITY (HSI)

- Describe colors by their hue, saturation and intensity.

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  - Saturation: Distance from center axis.
  - Intensity: Vertical axis.

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  - Hue: An angle from red (H = 0).
  - Saturation: Distance from center axis.
  - Intensity: Vertical axis.
- RGB is useful for color generation. HSI more useful for color description and color image processing.

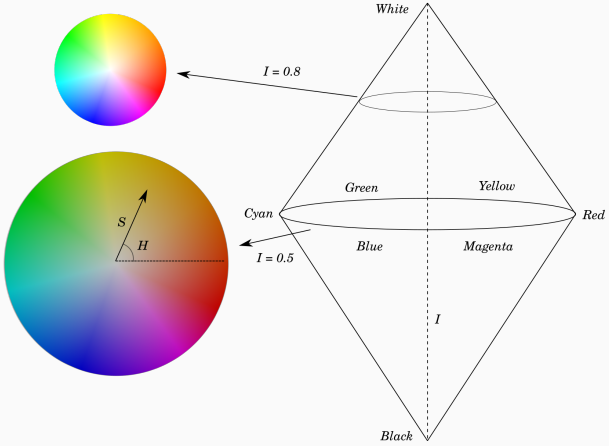

Figure 22: HSI double cone

True color uses all colors in the color space.

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- Used in applications that contain many colors with subtle differences. E.g. digital photography or photorealistic rendering.
- Two main ways to organize true color: Component ordering and packed ordering.

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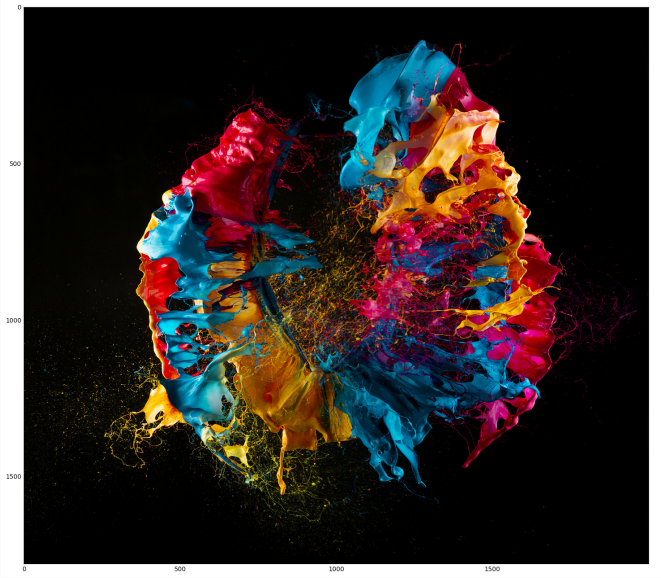
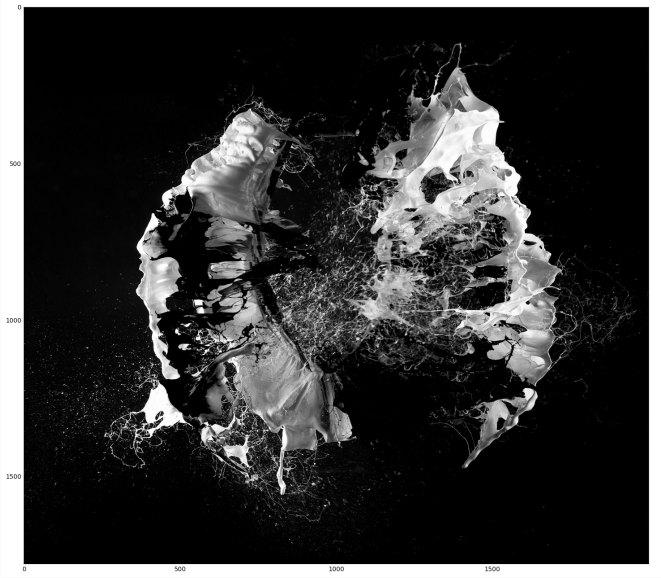
Indexed color uses only a subset of colors.

- Application dependence on which subset to use.
- Reduces memory and computation cost.
- Used when subtle color differences is not vital.

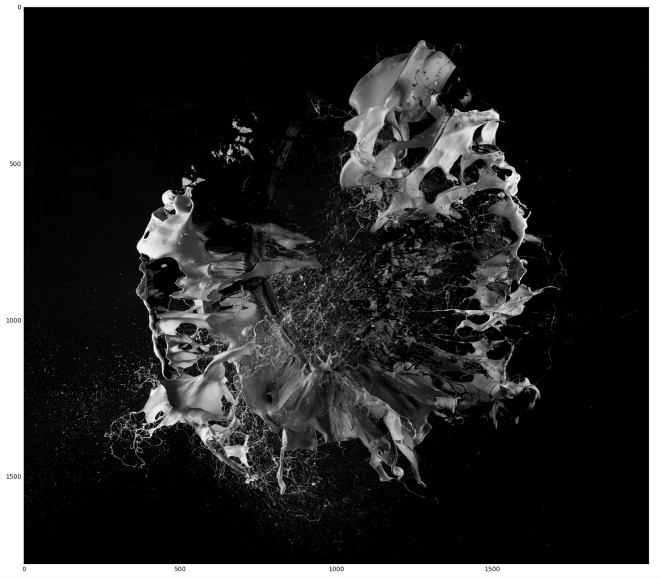
EXAMPLE



RED CHANNEL

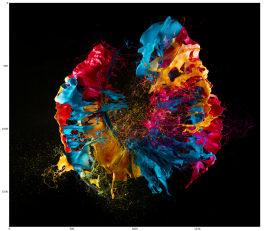


GREEN CHANNEL

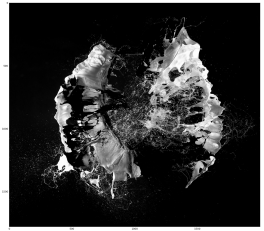


BLUE CHANNEL

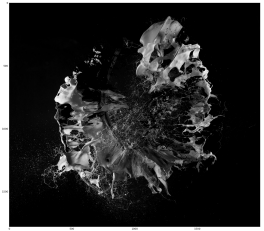
FULL EXAMPLE SUMMARY



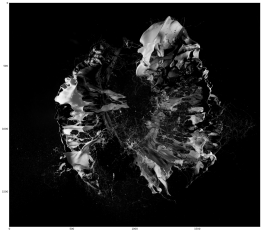
(a) All



(b) Red



(c) Green



(d) Blue

INDEXED COLOR

- An image contains indices of a look up table (LUT) of colors (palette) in stead of color intensity values.

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- Permits only a limited number of colors.
- Often are these images stored in indexed GIF or PNG formats.
- Use color quantization to transform optimally from a true color image to indexed color image.

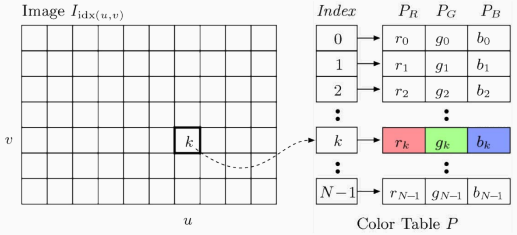

Figure 24: Indexed color ordering[1].

[1]Wilhelm Burger, Mark J. Burge, *Principles of Digital Image Processing: Fundamental Techniques*, 2010

COLOR IMAGES AND LOOK UP TABLES

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- Each row in the table represent a 24 bit RGB color.
- The table consist of the 256 colors that best represent the image.

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- Also often used in graphical display of data.
- If the color-LUT is mapped back to graylevels, the intensity should(!) be correct.
- Colormaps in plotting libraries are often pseudo-color LUT.

HOW TO CHOOSE THE CORRECT COLORMAP

- Often, we want the colormap to be *perceptually uniform*: Equal steps in the data are percieved as equal steps in the color space.

[1] See a more detailed wrap-up here: `https://bids.github.io/colormap/`

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- Up until recently, Matlab (until 2014) and Python's Matplotlib (until 2.0 release) used `jet` as the default colormap for data display[1].

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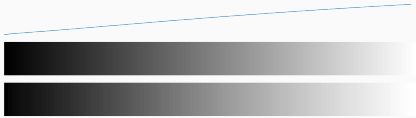
- Up until recently, Matlab (until 2014) and Python's Matplotlib (until 2.0 release) used `jet` as the default colormap for data display[1].
- `jet` is a rainbow colormap that is not perceptually uniform.

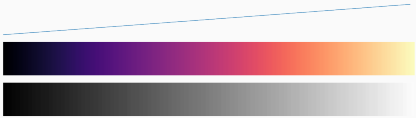
Figure 25: A gradient of `jet`, and the lightness component of the `CAM02-UCS` colorspace.

[1] See a more detailed wrap-up here: `https://bids.github.io/colormap/`

COMPARISON OF SOME COLORMAPS

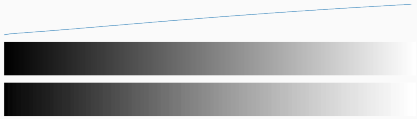


(a) `gray` (sequential)

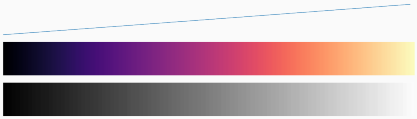


(b) `magma` (sequential perceptual uniform)

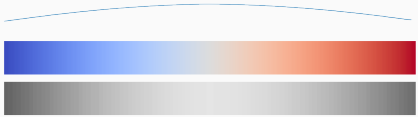
COMPARISON OF SOME COLORMAPS



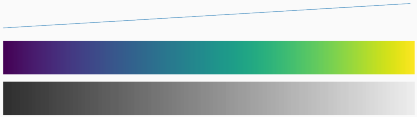
(a) `gray` (sequential)



(b) `magma` (sequential perceptual uniform)



(c) `coolwarm` (diverging)



(d) `viridis` (sequential perceptual uniform) (new matplotlib default)

Questions?